Grade 6 Mathematics, Quarter 4, Unit 4.1

Applications of Equations and Inequalities

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Understand solving an equation or inequality as a process of answering a question from a specified set.
- Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- Write an inequality to represent a constraint or condition in a real-world or mathematical problem.
- Recognize that inequalities have infinitely many solutions.
- Represent solutions on number line diagrams.
- Use variables to represent two quantities in a real-world problem that change in relation to one another.
- Write an equation to express the dependent variable.
- Analyze the relationship between the dependent and independent variables using graphs and tables.
- Relate relationships between dependent and independent variable to the equation.
- Fluently divide multidigit numbers using the standard algorithm.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Communicate why equations are inequalities.
- Communicate how to make an equation or inequality a true statement.

Model with mathematics.

- Represent solutions of inequalities on a number line.

Attend to precision.

- Understand distributive property and use it to check for correct answers. (i.e., reciprocal operations.)
- Fluently utilize a variety of methods to problem solve and check work for correct answers.
- Divide multidigit numbers accurately.

Essential questions

- How do you solve an equation or inequality as a process of answering a question from a specified set?
- How can you use substitution to determine whether a given number in a specified set makes an equation or inequality true?
- How do changes in two quantities represent the relationship between variables?
- How do you write an equation to express the dependent variable?
- How can you use graphs and tables to analyze

- How do you write an inequality to represent a constraint or condition in a real-world or mathematical problem?
- How do you recognize that inequalities have more than one solution?
- How do you plot solutions on number line diagrams?
- How do you represent the relationship between the dependent and independent variables?
- What is the relationship between dependent and independent variables in an equation?
- How do you divide multidigit numbers using a standard algorithm?

Written Curriculum

Common Core State Standards for Mathematical Content

Expressions and Equations	**6.EE**

Reason about and solve one-variable equations and inequalities.

6. EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6. EE.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

The Number System	**6.NS**

Compute fluently with multi-digit numbers and find common factors and multiples.

6. NS.2 Fluently divide multi-digit numbers using the standard algorithm.

Common Core Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In fourth grade, students learned to add, subtract, multiply, and divide two-digit numbers using standard algorithms. In fourth grade, students fluently added and subtracted multidigit whole numbers and multiplied whole numbers up to four digits by one digit.

In fifth grade, students fluently multiplied and divided multidigit whole numbers with decimals to hundredths. They also used parentheses, brackets, or braces in numerical expressions and evaluated expressions with symbols.

Current Learning

In the first nine weeks of sixth grade, students develop standard algorithms for dividing multidigit numbers.

Students fluently develop standard algorithms for dividing multidigit numbers. They use expressions and equations fluently.

Students solve an equation or inequality as a process of answering a question.

Students write inequalities for real-world situations and recognize that inequalities have an infinite number of solutions.

Future Learning

Students will develop an understanding of operations with rational numbers and working with expressions and linear equations.

Additional Findings

In the PARCC *Expressions and Equations* document, it states, "It is important for students to state precisely the meaning of variables they use when setting up equations." (p. 7)

According to *Adding It Up*, "relatively little research is available to shed light on how students think about multi-digit division or what learning activities might be of more help to them." (p. 210)

Grade 6 Mathematics, Quarter 4, Unit 4.2

Recognizing and Displaying Numerical Data

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Recognize statistical questions that anticipate variability in data questions.
- Recognize the measure of center for a numerical data set.
- Understand that a data set summarizes all of its values with a single number.
- Recognize that a measure of variation describes how its values vary with a single number.
- Use a number line to display numerical data.
- Use dot plots to display numerical data.
- Use histograms to display numerical data.
- Use box plots to display numerical data.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Communicate the findings of statistical data.
- Reason inductively about data.

Use appropriate tools strategically.

- Use models to a display data.

Attend to precision.

- Understand distributive points of data and their relationship to a data set.
- Fluently utilize a variety of methods to interpret data.

Essential questions

- How can you use statistical questions to anticipate variability?
- How do you find the measure of center for a numerical data set?
- How can a data set summarize all of its values with a single number?
- How can you show variation in a data set with a single number?
- How can you make a number line to display numerical data?
- How can you make a dot plot to display numerical data?
- How can you make a histogram to display numerical data?
- How can you make a box plot to display numerical data?

Written Curriculum

Common Core State Standards for Mathematical Content

Statistics and Probability	**6.SP**

Develop understanding of statistical variability.

6. SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages*.

6. SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

6.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In third grade, students drew a scaled picture graph and bar graph to represent data. In fifth grade, students made line plots to display data sets.

Current Learning

Students recognize the numerical data set as a summary of all values involved with discussions on values in single number opposed to data sets values.

Future Learning

In seventh grade, students will use random sampling to draw inferences about a population and draw informal comparative inferences about two populations.

Additional Findings

According to the *PARCC Statistics and Probability* document, "Statistical investigations begin with a question and students now see that answers to such questions always involve variability in the data collected to answer them." (p. 4)

According to *Adding It Up,* "Research in statistics and probability is less extensive than that in number and operation, in algebra, or in measurement and geometry." (p. 288)

Grade 6 Mathematics, Quarter 4, Unit 4.3

Analyzing Numerical Data

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Understand the center, spread, and overall shape of a data set.
- Report numbers of observation as a summary of a data set.
- Describe the units of measurement when summarizing numerical data.
- Describe quantitative measures using the terms median, mean, interquartile range, mean absolute deviation within a pattern (in context of data collection), and striking deviations.
- Relate the shape and context of data collected to the choice of measures and variability.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Use models of data collection appropriately.
- Detect possible errors by strategically using estimation and other mathematical knowledge.

Attend to precision.

- Choose appropriate distribution model to represent data.
- Be careful about specifying units of measure, and labeling axes to clarify the correspondence of quantities in a problem.

Essential questions

- What does the center, spread, and overall shape of a data set represent?
- How can you summarize numbers of observation as a data set?
- How can you summarize a numerical data set using appropriate units of measurement?
- How do you find the median?
- How do you find the mean?
- How do you find the interquartile range?
- How do you find the mean absolute deviation within a pattern in context of data collection?
- How does the shape and context of data collected relate to the choice of measures and variability?

Written Curriculum

Common Core State Standards for Mathematical Content

Statistics and Probability	6.SP

Develop understanding of statistical variability.

6. SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

Summarize and describe distributions.

6. SP.5 Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations.

b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

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Clarifying the Standards

Prior Learning

In first grade, students organized, represented, and interpreted data with up to three categories.

Current Learning

Students understand that data collection can be described through distribution by its center, spread, and overall shape.

Students can report data as a number of observations and by the nature of attributes, including units of measurement.

Students can define median, mean, variability, interquartile range, and absolute deviation and recognize patterns in a data set.

Future Learning

In seventh grade, students will use data to compare two populations and will be introduced to probability of outcomes in terms of data. Students move from concentrating on analysis of data to production of data.

Additional Findings

According to the *PARCC Statistics and Probability* document, "[students] identify clusters, peaks, and gaps, recognizing common shapes and patterns in these displays of data distributions. A major focus of grade 6 is characterization of data distributions by measures of center and spread." (p. 4)

According to *Adding It Up,* "Some students consider the average to be a data point roughly centered within the data, that is, they conceptualize average as median." (p. 290)